last available order must be computed intellectually, and then, indeed, we might set up the type by machinery, provided each successive difference were set by hand. The instrument would therefore fall to be classed among composing machines.

Thus, on the whole, arithmeticians have not much to expect from the aid of calculating machines. A few tables, otherwise very easily made, comprise the whole extent of our expected benefits; and we must fall back upon the wholesome truth that we cannot delegate our intellectual functions, and say to a machine, to a formula, to a rule, or to a dogma, I am too lazy to think, do please think for me.

[The author overlooks the advantage of the arithmometer when used in a very long series of calculations, namely, that the work is almost entirely mechanical, and in consequence much less fatiguing after a very moderate degree of use, than direct calculation, which requires a greater mental strain. This was remarked to us some years ago, by Dr. Zillmer, President of the German Life Insurance Institute, who has used the machine extensively, and who has described some of its applications in a paper of which a translation is printed in the number of this *Journal* for April, 1869, p. 25.—Ed. *J. I. A.*]

Mr. W. J. Hancock, Actuary and Secretary of the Patriotic Assurance Company, who has had much experience in the use of the Arithmometer, has favoured us with the following observations, in further elucidation of the points discussed in the preceding papers of Major-General Hannyngton and Mr. Sang:—

In the foregoing papers by Major-General Hannyngton and Mr. Sang we have a very full description of M. Thomas' arithmometer, but there appears to be a difference of opinion as to its practical utility.

Major-General Hannyngton's recommendation is based on a lengthened practical experience of its working.

Mr. Sang, while stating that on the whole arithmeticians have not much to expect from the aid of calculating machines, does not appear to have used it to any extent.

It is admitted that the machine does the first four rules of arithmetic correctly, and the question is—does it save time, risk of error and mental labour? Take the case of multiplication; we have three methods open to us.

1st. By direct multiplication of the multiplicand by the multiplier, figure by figure.

2nd. By the manipulation of certain other numbers, which bear known relations to the numbers we wish to multiply together, such as logarithms and quarter squares.

3rd. By mechanical aid, such as the slide rule and the arithmometer.

If we take 8 figures to be multiplied by 8 figures—

By the 1st method we have 64 separate multiplications; we have to consider 64 to 72 times what is to be carried, to write down from 64 to 72 figures; then to make 14 additions, some of 8 figures each, and to consider 14 or 15 times what is to be carried.

By the 2nd method the logarithms of the multiplicand must be sought out, and the proportional parts for those figures not given directly in the table used taken out, and the result written down. The same must be done for the multiplier. The numbers so obtained must then be added, and the resulting logarithms converted into the answer required.

By the 3rd method, using the arithmometer, the multiplicand is put on the lower plate or face of the machine; the attention of the operator is then only required to turn the handle the proper number of times, and shift the slide one step for each figure in the multiplier. If on inspection it be found that the figures on the lower plate and on the multiplier disks be correct, the product shown on the upper disks must be correct, unless the machine be broken in some part.

In using the machine we may consider placing the buttons on lower plate as being equal to writing the multiplicand on a piece of paper, and the turning of the handle and shifting the slide to produce the multiplier as a little more than writing the multiplier under the multiplicand. Then we save the mental labour and loss of time incident to long multiplication. The risk of error in the machine is reduced to the risk that some one or more of the figures in the multiplicand and multiplier are wrong. In long multiplication there is, in addition to this risk (a figure may be written on paper wrong as well as be put on the machine wrong), the risk that some of the numerous mental operations to which I have referred may be wrong.

I think it is therefore manifest that the machine is far superior to the long multiplication by hand, when 8 figures by 8 figures are involved, and that it saves mental labour, time and risk of error. Of course the value of the machine diminishes with the number of figures required to be dealt with, and is not perhaps marked when the figures in multiplicand or multiplier do not exceed two.

If a man has not half a mile to travel, there is not much difference between walking and going in a railway train; but when the distance is one or two hundred miles, the advantage of going by train instead of walking becomes evident.

In comparing the machine with the use of logarithms there are two things to be considered:—First, the mental labour and risk of error in finding out the logarithms of multiplicand and multiplier, in their addition, and then in finding the number corresponding to the resulting logarithms. The mental labour here is proportionately greater for the total number of figures used in the operation than in the case of long multiplication. Second, there are many persons who are neither actuaries nor computers who do not know the use of logarithms, and yet have to multiply and divide large figures.

With regard to division the assistance afforded by the machine is, other things equal, only a little less than in the case of multiplication. I cannot agree with Mr. Sang that the mental fatigue of watching the quotient, when working the machine, is greater than in long division. Should the handle be turned once too often, it is at once detected by seeing one or two 9's on the left of the dividend on the slide, and this can be

rectified in an instant by putting the machine to addition, when one turn of the handle corrects the error.

The difference between multiplication and division, when performed by logarithms, being only the difference between adding and subtracting the logarithms, the advantage of the machine for division, so far as time and mental labour are concerned, is not quite so great as for multiplication; but, with regard to risk of error, the advantage for both multiplication and division is the same.

Mr. Sang's objections appear to amount to about three.

First. That a computer is far more liable to make a mistake in copying than commit an error in calculation.

Assuming that he is correct in this statement—which I do not admit—I think it is evident that when more than a few figures are dealt with, the computer has far more opportunities of committing an error in calculation than of making a mistake in copying.

Second. That short cuts and abbreviations occurring in general practice are lost in machine work. It is no doubt true that in many classes of cases the quantities to be dealt with bear such a relation to each other that short cuts are of great use. A butcher's boy will sometimes be able to tell how much so many pounds of beef, at so much a pound, will amount to, in a time that would astonish very able computers. Major-General Hannyngton has pointed out in his paper that the machine has short cuts and methods of its own, and that from his experience the machine requires special formulas; and Dr. Zillmer has pointed out the same thing.

The following example of a decreasing annuity calculation will illustrate the use of the machine.

Carlisle $3\frac{1}{2}$ percent (Chisholm):—

(24.) $\dfrac{(a+b)\mathrm{N}_{x+1|n}-b(\mathrm{S}_{x+1|n}-n\mathrm{N}_{x+1+n})}{\mathrm{D}_x}$ =the present value of an annuity for n years, commencing at £a and decreasing by £b annually until the end of the term.

$x=35,\ x+1=36,\ n=23$

$x+1+n=59$

$$a=\begin{cases} 16{\cdot}525 & \text{Instalment} \\ 15{\cdot}203 & \text{Interest on £380·075 at 4 percent} \end{cases}$$

$$\underline{\qquad}$$

$$31{\cdot}728$$

b ·661 Interest on instalment at 4 percent

$$\underline{\qquad}$$

$a+b$ 32·389 At 4 percent.

$\mathrm{N}_{36}=27615{\cdot}6$
$\mathrm{N}_{59}=\ 5608{\cdot}3$

$$\underline{\qquad}$$

$22007{\cdot}3=\lambda 4{\cdot}3425667$

$32{\cdot}389\ =\lambda 1{\cdot}5103975$

$$\underline{\qquad}$$

$5{\cdot}8529642=712794$

S_{36}=397165·
S_{59}= 50314·

346851·
$23N_{59}$ 128991

217860 =λ5·3381775
·661 λ$\bar{1}$·8202415

5·1583790=144005

N_{59} 5608·3
23

16824·9
112166

128990·9

568789=λ5·7549511

D_{35}=1608·48=λ3·2064159

2·5485352=353·6185.

By machine:—

$a+b$ 32·389 ×
$N_{36}-N_{59}$=22007·3 } . . . . =712794
b . . . . . =·661 ×
$S_{36}-S_{59}$=346851· −
23 ×
N_{59} =128991· } =217860 } =144005· } = 568789 } 353·6189
÷
D_{35} . . . . . 1608·48 }

The calculation is done as follows:—N_{59} is put on the face of the machine, which is set for multiplication, the handle is turned three times, the slide shifted one step to the right, the handle is then turned twice and 128991 is read from the upper holes and recorded, and the slide effaced. S_{36} is put on the face, the handle turned once and it appears in the upper holes. S_{59} is then put on the face, the machine set for subtraction, and the handle turned once, and 346851 appears in the upper holes. 128991 is now put on the face, the handle turned once, 217860 appears in the upper holes. This number is then put on the face, the slide effaced, the machine set for multiplication, the handle turned once, the slide shifted one step, the handle turned six times, the slide shifted one step, the handle turned six times, and 144005· appears in the upper holes and is recorded. The slide being again effaced, N_{36} is put on the face, the machine set for addition, the handle turned once, and it appears in the upper holes. N_{59} is then put on the face, the machine set for subtraction, the handle turned once, 22007·3 appears in the upper holes. This number is then put on the face, the slide effaced, the machine set for multiplication, the handle turned nine times, the slide shifted one step, the handle turned eight times, and so on, until 32·389 appears on the multiplier disks, then 712794 will appear in the upper holes. 144005 is now put on the face, the machine set for subtraction, the handle turned once, and 568789 appears in the upper holes; the slide can be so placed in the multiplication of $(a+b)N_{x+1|n}$ that these figures will be in the left-hand holes of the slide—otherwise they must be

set up in those holes. 1608·48 is then set on the face, the machine set for division, the quotient disks effaced, the handle turned three times, the slide shifted one step, the handle turned five times, and so on until 353·6189 appears in the quotient holes. If the reciprocal of 1608·48 be taken, then 568789 is put on the face, and multiplication is performed until that reciprocal appears in the multiplier holes; then 353·6189 will appear in the upper holes. This calculation is made on a six-figure machine, so that the last figure taken down often has the usual increase of 1 when the next figure is 5 or upwards. The time taken to turn the handle and shift the slide is almost instantaneous. The decimal points are regulated by the ivory pins.

Thirdly. That the use of the machine will not make us good computers. This objection is exactly of the same nature as the objection often made to the use of machinery in place of hand labour.

In an interesting lecture delivered by the Rev. Professor Haughton, F.T.C.D., M.D., at the Royal Institution of Great Britain, on 27th May, 1871, he points out that the human muscle, like the planet moving in its orbit, or the bee making its cell, performs its work on the principle of least action. I do not see why the brain should not do its work on the same principle; so that if the arithmometer can do those calculations for which it is suited, in less time, or in the same time, but with less mental labour than the hand alone can do them, it should be used. We do not expect the calculating machine to think for us, but to save the brain in doing mechanical work.

During the very interesting discussion which followed the reading of General Hannyngton's paper at the Institute of Actuaries, every gentleman who spoke of having used the machine for any length of time referred to the saving of time and mental labour it effected. Mr. Sprague pointed out that logarithms would be more convenient where three, four, or more factors were to be multiplied together; or where several factors were to be divided by two or three others.

No doubt there are many calculations in which the machine would not give assistance; but that does not make it less useful where it can be applied. A sewing machine is not condemned because it will not sew on buttons. On the whole, I think the balance of argument is in favour of the arithmometer; and I have no doubt that with more extensive use improvements will be made in construction and workmanship.

On the Equitable Apportionment of a Fund between the Life Tenant and the Reversioner. *By* A. Baden, *Fellow of the Institute of Actuaries.*

[Read before the Institute, 27th March, 1871.]

If any excuse be wanting for once more bringing this question forward for your consideration, it is not because the question itself is unimportant. The frequency of the cases involving it upon which the opinion of actuaries is sought, and the magnitude of the